Superfield actions for N=8 and N=6 conformal theories in three dimensions

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ABSTRACT: The manifestly supersymmetric pure spinor formulations of the Bagger-Lambert-Gustavsson models with $N = 8$ supersymmetry and the Aharony-Bergman-Jafferis-Maldacena models with $N = 6$ supersymmetry are given. The structures of the pure spinors are investigated in both cases, and non-degenerate measures are formed using non-minimal sets of variables, allowing for the formulation of an action principle.

KEYWORDS: Extended Supersymmetry, Superspaces, Conformal Field Models in String Theory, BRST Symmetry.

There has recently been much interest in conformal three-dimensional theories. Following the discovery of the existence of a maximally supersymmetric ($N = 8$) interacting theory of scalar multiplets coupled to Chern-Simons, the Bagger-Lambert-Gustavsson (BLG) theory [1 – 4], much effort has been spent on trying to generalise the construction and to interpret it in terms of an AdS boundary model of multiple M2-branes. The interesting, but restrictive, algebraic structure of the model, containing a 3-algebra with antisymmetric structure constants, turned out to have only one finite-dimensional realisation [5, 6], possible to interpret in term of two M2-branes [7, 8] (see however refs. [9, 10] dealing with the infinite-dimensional solution related to volume-preserving diffeomorphisms in three dimensions).

It then became an urgent question how the stringent requirements in the BLG theory could be relaxed. There are different possibilities. One may let the scalar product on the matter representation be degenerate [11]. This works at the level of equations of motion, but does not allow for an action principle. One may also go one step further, and add further null directions to that degenerate case, which leads to scalar products with indefinite signature [12 – 14] (and consequently to matter kinetic terms with different signs). Or, finally, one may reduce the number of supersymmetries, specifically to $N = 6$, as proposed by Aharony, Bergman, Jafferis and Maldacena (ABJM) [15], or maybe even to lower N [16, 17]. The $N = 6$ models were further studied in refs. [18 – 24] (among other papers). For recent developments in the theory of multiple membranes, we refer to ref. [25] and references given there. The literature on the subject is huge, and we apologise for omissions of references to relevant papers.

The superfield formulation of the BLG model was given in our previous paper [26] (see also ref. [27], where the on-shell superfields were constructed for the example of the BLG model based on the infinite-dimensional algebra of volume-preserving diffeomorphisms in three dimensions). A superfield formulation with $N = 1$ superfields was given in ref. [28] and with $N = 2$ superfields in ref. [29]. In ref. [26] we constructed an action in an $N = 8$ pure spinor superspace formulation of the BLG model, which covers all situations with $N = 8$ above except the ones with degenerate scalar product. The purpose of the present paper is twofold. Firstly, we construct the corresponding formulation for $N = 6$ superfields, thus covering the ABJM models. These $N = 6$ models are of course not maximally supersymmetric, but still more than half-maximally, so the component actions have only on-shell supersymmetry, which means that appropriate pure spinors are needed. Secondly, in ref. [26], some aspects of the measure on non-minimal pure spinor space were left out, and simply assumed to work in a similar way as in $D = 10$. Here, we remedy this omission by analysing the pure spinor constraints, adding non-minimal variables in the spirit of ref. [30] and forming explicit non-degenerate measures, both for $N = 8$ and $N = 6$, thus completing the construction of manifestly supersymmetric actions for the BLG and ABJM models, which can hopefully be used to improve on quantum calculations [31, 32].

Let us first briefly review the results of ref. [26]. Since the BLG model is maximally supersymmetric, component formulations and also usual superspace formulations are on-shell. There is no finite set of auxiliary fields. A pure spinor treatment is necessary in order to write an action in a generalised BRST setting. (For the use of pure spinors and pure spinor superspaces in string theory we refer to refs. [33, 34, 30], and in field theory to

gh# =	1	0	−1	−2	−3
dim = 0	(0)(0000)				
$\frac{1}{2}$	•	•			
1	•	(2)(0000)	•		
$\frac{3}{2}$	•	•	•	•	
2	•	•	(2)(0000)	•	•
$\frac{5}{2}$	•	•	•	•	•
3	•	•	•	(0)(0000)	•
$\frac{7}{2}$	•	•	•	•	•

Table 1: The cohomology of the scalar complex.

refs. [35 – 46].) The Lorentz algebra in $D = 3$ is $so(1,2) \approx sl(2,\mathbb{R})$. The $N = 8$ theory has an $so(8)$ R-symmetry, and we choose the fermionic coordinates and derivatives to transform as $(\mathbf{2}, \mathbf{8}_s) = (1)(0010)$ under $sl(2) \oplus so(8)$. This representation is real and self-conjugate. The pure spinors transform in the same representation, and are written as $\lambda^{A\alpha}$, where A is the $sl(2)$ index and α the $so(8)$ spinor index. As usual, a BRST operator is formed as $Q = \lambda^{A\alpha} D_{A\alpha}$, D being the fermionic covariant derivative. The nilpotency of Q demands that

$$(\lambda^A \lambda^B) = 0 \ , \tag{1}$$

where $(\ldots)$ denotes contraction of $so(8)$ spinor indices, since the superspace torsion has to be projected out. This turns out to be the full constraint.[1] These pure spinors are similar to those encountered in ref. [47]. The "pure spinor wave function" for the Chern-Simons field is a fermionic scalar Ψ of (mass) dimension 0 and ghost number 1. For the matter multiplet we have a bosonic field Φ^I in the $so(8)$ vector representation $(0)(1000)$ of dimension $1/2$ and ghost number 0. In addition to the pure spinor constraint, the matter field is identified modulo transformations

$$\Phi^I \to \Phi^I + (\lambda^A \sigma^I \varrho_A) \tag{2}$$

for arbitrary ϱ. In this minimal pure spinor formulation the fields are expanded in power series in λ, i.e., in decreasing ghost number. The field content (ghosts, fields and their antifields) are read off from the zero-mode BRST cohomology given in tables 1 and 2 for the Chern-Simons and matter sectors respectively.

We observe that the field content is the right one. In Ψ we find the ghost, the gauge connection, its antifield and the antighost. The antifield has dimension 2 (as opposed to e.g. $D = 10$ super-Yang-Mills, where it has dimension 3), indicating equations of motion that are first order in derivatives. In Φ we find the eight scalars ϕ^I, the fermions $\chi^{A\dot\alpha}$ and their antifields. In addition, the field Ψ transforms in the adjoint representation **adj** of some gauge group and Φ^I in some representation **R** of the gauge group. The corresponding indices are suppressed.

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[1] The vanishing of the "torsion representation" — the vector part of the spinor bilinear — is necessary, but does not always give the full pure spinor constraint. One example where further constraints are needed is $N = 4$, $D = 4$ super-Yang-Mills theory.

gh# =	0	−1	−2	−3	−4
dim $= \frac{1}{2}$	(0)(1000)				
1	(1)(0001)	•			
$\frac{3}{2}$	•	•	•		
2	•	(1)(0001)	•	•	
$\frac{5}{2}$	•	(0)(1000)	•	•	•
3	•	•	•	•	•
$\frac{7}{2}$	•	•	•	•	•

Table 2: The cohomology of the vector complex.

In order to derive the equations of motion for the physical component fields, one starts from the ghost number zero part of the fields (i.e., $\Phi^I \to \phi^I(x,\theta)$ and $\Psi \to \lambda^\alpha A_\alpha(x,\theta)$ respectively), and examines the content of the θ expansion by repeated application of fermionic covariant derivatives, using the pure spinor constraint and the reducibility (2) when they occur. As a guideline one has the cohomology at ghost number one; these representations are the only ones where an equation of motion may sit, for obvious reasons. In this manner, one derives the linearised component equations $\Box\phi^I = 0$, $\partial\!\!\!/\chi^A = 0$ for the scalar multiplet, and $dA = 0$ for the Chern-Simons field, and also the interacting equations from the actions below.

In ref. [26], it was assumed that a non-degenerate measure can be formed using a non-minimal extension of the pure spinor variables along the lines of ref. [30]. This measure, including the three-dimensional integration, should carry dimension 0 and ghost number -3, and should allow "partial integration" of the BRST charge Q. It was then shown that the Lagrangian of the interacting model is of a very simple form, containing essentially a Chern-Simons like term for the Chern-Simons field, minimally coupled to the matter sector:

$$\mathcal{L} = <\Psi, Q\Psi + \frac{1}{3}[\Psi,\Psi]>_{\mathbf{adj}} + \frac{1}{2}M_{IJ}<\Phi^I, Q\Phi^J + \Psi \cdot \Phi^J>_{\mathbf{R}} \ . \tag{3}$$

The brackets denote (non-degenerate) scalar products on $\mathbf{adj}$ and $\mathbf{R}$, $[\cdot,\cdot]$ the Lie bracket of the gauge algebra and $T \cdot x$ the action of the Lie algebra element in the representation $\mathbf{R}$. M_{IJ} is the pure spinor bilinear $\varepsilon_{AB}(\lambda^A\sigma_{IJ}\lambda^B)$, which is needed for several reasons: in order to contract the indices on the Φ's antisymmetrically, to get a Lagrangian of ghost number 3, and to ensure invariance in the equivalence classes defined by eq. (2).

The invariances of the interacting theory, generalising the BRST invariance in the linearised case, are:

$$\delta\Psi = Q\Psi - [\Lambda,\Psi] - M_{IJ}\{\Phi^I, \Xi^J\} \ ,$$
$$\delta\Phi^I = -\Lambda \cdot \Phi^I + (Q + \Psi\cdot)\Xi^I \ , \tag{4}$$

where Λ is an adjoint boson of dimension 0 and ghost number 0, and Ξ^I a fermionic vector in $\mathbf{R}$ of dimension 1/2 and ghost number -1. Here we also introduced the bracket $\{\cdot,\cdot\}$ for the formation of an adjoint from the antisymmetric product of two elements in $\mathbf{R}$, defined via $<x, T \cdot y>_{\mathbf{R}} = <T, \{x,y\}>_{\mathbf{adj}}$. The invariance with parameter Λ is manifest.

The transformation with Ξ has to be checked. One then finds that the transformation of the matter field Φ gives a "field strength" contribution from the anticommutator of the two factors $Q + \Psi$, which is cancelled against the variation of the Chern-Simons term. The single remaining term comes from the transformation of the Ψ in the covariant matter kinetic term, and it is proportional to $M_{IJ}M_{KL}<\{\Phi^I, \Phi^J\}, \{\Phi^K, \Xi^L\}>_{\mathbf{adj}}$. Due to the pure spinor constraint, $M_{[IJ}M_{KL]} = 0$. This was shown in ref. [26], using the simple observation that the only $sl(2)$ singlet at the fourth power of λ is in the $sl(2) \oplus so(8)$ representation $(0)(0200)$ — the four-index antisymmetric tensors $(0)(0020)$ or $(0)(0002)$ do not occur. So if the structure constants of the 3-algebra defined by $<\{x, a\}, \{b, c\}>_{\mathbf{adj}} = <x, [\![a, b, c]\!]>_{\mathbf{R}}$ are antisymmetric, this term vanishes. It was also checked that the commutator of two Ξ-transformations gives a Λ-transformation together with a transformation of the type (2).

Having thus reviewed the results of ref. [26], we would like to do the corresponding construction for $N = 6$. The R-symmetry now is $so(6) \approx su(4)$. We use $A_1 \oplus A_3$ notations for Dynkin labels. The twelve supercharges are in the (quasi-real) representation $(1)(010)$. The four complex scalar fields should come in $(0)(100)$ (and their conjugates in $(0)(001)$). The pure spinor[2] is $\lambda^{A\alpha\beta} = -\lambda^{A\beta\alpha}$, where $A = 1, 2$, $\alpha, \beta = 1, \ldots 4$. Later we will equivalently write λ with an $so(6)$ vector index as λ^{Ai} The general symmetric product of two "spinors" is $\oplus_s^2 (1)(010) = (0)(101) \oplus (2)(000) \oplus (2)(020)$. The second of these represents the torsion. We will need to keep the first one for writing the matter lagrangian. The pure spinor constraint is simply $\varepsilon_{\alpha\beta\gamma\delta}\lambda^{A\alpha\beta}\lambda^{B\gamma\delta} = 0$, or equivalently

$$\lambda^{Ai}\lambda^{Bi} = 0 \ . \tag{5}$$

It has the same formal structure as in the $N = 8$ case, only that λ is an $so(6)$ vector instead of an $so(8)$ vector (after triality rotation).

A scalar wave function has "the same" cohomology as in ref. [26] (in table 1, just replace $(n)(0000)$ under $sl(2) \oplus so(8)$ with $(n)(000)$ under $sl(2) \oplus su(4)$). So Chern-Simons is described in a formally identical manner. The matter multiplet comes as expected from a bosonic wave function Φ^α in $(0)(100)$ and in the equivalence class

$$\Phi^\alpha \approx \Phi^\alpha + \lambda^{A\alpha\beta}\varrho_{A\beta} \ . \tag{6}$$

The cohomology is the right one, shown in table 3.

The field Φ^α transforms in some representation $\mathbf{R}$ of the gauge group, and $\bar\Phi_\alpha$ in $\bar{\mathbf{R}}$. The matter Lagrangian must again contain two powers of λ through the combination $M_\alpha{}^\beta = \frac{1}{2}\varepsilon_{AB}\varepsilon_{\alpha\gamma\delta\varepsilon}\lambda^{A\beta\gamma}\lambda^{B\delta\varepsilon}$, which is exactly the $(0)(101)$. We write the Lagrangian:

$$\mathcal{L} = <\Psi, Q\Psi + \frac{1}{3}[\Psi, \Psi]>_{\mathbf{adj}} + M_\alpha{}^\beta <\Phi^\alpha, (Q + \Psi \cdot)\bar\Phi_\beta>_{\mathbf{R} \otimes \bar{\mathbf{R}}} \tag{7}$$

with obvious notation. The generalised BRST invariance now reads

$$\begin{aligned}\delta\Psi &= Q\Psi - [\Lambda, \Psi] - M_\alpha{}^\beta\{\Phi^\alpha, \bar\Xi_\beta\} - M_\alpha{}^\beta\{\Xi^\alpha, \bar\Phi_\beta\} \ , \\ \delta\Phi^\alpha &= -\Lambda \cdot \Phi^\alpha + (Q + \Psi \cdot)\Xi^\alpha \ ,\end{aligned} \tag{8}$$

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[2]The representation of the fermionic derivatives and of the λ's are of course not spinor representations of the R-symmetry group, only of the Lorentz group. For convenience, we stick to the terminology "spinor" and "pure spinor" also in this case.

gh# =	0	−1	−2	−3	−4
dim = $\frac{1}{2}$	(0)(100)				
1	(1)(001)	•			
$\frac{3}{2}$	•	•	•		
2	•	(1)(001)	•	•	
$\frac{5}{2}$	•	(0)(100)	•	•	•
3	•	•	•	•	•
$\frac{7}{2}$	•	•	•	•	•

Table 3: The cohomology of the $N = 6$ matter complex.

The "critical term", as in the $N = 8$ case, is the one that transforms the Ψ in the matter Lagrangian under the matter gauge transformation. One gets a term proportional to

$$M_\alpha{}^\gamma M_\beta{}^\delta <\{\Phi^\alpha, \bar{\Phi}_\gamma\}, \{\Phi^\beta, \bar{\Xi}_\delta\} + \{\Xi^\beta, \bar{\Phi}_\delta\}>_{\mathbf{adj}} . \qquad (9)$$

Now, the tensor $N_{\alpha\beta}{}^{\gamma\delta} = M_\alpha{}^\gamma M_\beta{}^\delta$ turns out to be traceless and symmetric in $(\alpha\beta)$ and in $(\gamma\delta)$, i.e., it transforms in the 84-dimensional representation $(0)(202)$. This is the only $so(1,2)$ scalar at λ^4 due to the pure spinor constraint. This gives a weaker condition on the structure constants of the "3-algebra" than in the $N = 8$ case: antisymmetry in pairs [22], apart from the structure already assumed. The classification of such algebraic structures was performed in ref. [23]. It is satisfactory that the structure of the pure spinors in both cases give the necessary and sufficient algebraic structure by the vanishing of a single term in the transformations.

In ref. [26], only the minimal pure spinors were considered, and in practice regarded only as a book-keeping device through the expansion in powers of λ. The existence of a non-minimal extension of the variables along with a non-degenerate measure was assumed in order that the action should be well-defined. We will now analyse the pure spinor constraints for the $N = 8$ and $N = 6$ pure spinors, add non-minimal variables and show how the non-degenerate measures of correct dimension and ghost number arise.

The measure is associated with the singlet cohomology of the antighost in the Chern-Simons complex. With minimal pure spinor variables, one may prescribe that this component of an integrand is picked out, like a residue. Picking out a component at $\lambda^3\theta^3$ gives dimension 3 and ghost number -3, and together with the three-dimensional x-integration dimension 0 and ghost number -3. This goes well together with the Lagrangians above having dimension 0 and ghost number 3. Such a "measure" is however degenerate, and can not be used to form the actions, due to the fact that the fields are expanded in positive powers of λ only.

A remedy, based on the analogous construction in $D = 10$ [30], is to introduce further variables. Not only does the new measure become non-degenerate, it is also defined in terms of full integrals over all variables, including the θ's. Let us recall the 10-dimensional construction. In addition to the pure spinor λ^α with the constraint $(\lambda\gamma^a\lambda) = 0$, one has another bosonic pure spinor μ_α, with $(\mu\gamma^a\mu) = 0$, of opposite chirality, and a fermionic spinor r_α fulfilling $(\mu\gamma^a r) = 0$. We denote the canonically conjugate variables (derivatives)

	gh#	dim
$d^D x$	0	$-D$
$d^{16}\theta$	0	8
$[d\lambda]$	8	-4
$[d\mu]$	$8\,\mathrm{gh\#}(\mu)$	$8\,\dim(\mu)$
$[dr]$	$-11 - 8\,\mathrm{gh\#}(\mu)$	$-8\,\dim(\mu)$
total	-3	$-(D-4)$

Table 4: The dimensions and ghost numbers of the $D = 10$ measure.

to μ_α and r_α by u^α and s^α, respectively. The new BRST operator is $\mathcal{Q} = \lambda^\alpha D_\alpha + u^\alpha r_\alpha$, and its cohomology is independent of μ and r. Let μ have dimension $\dim(\mu)$ and ghost number $\mathrm{gh\#}(\mu)$. Then r has dimension $\dim(\mu)$ and ghost number $1 + \mathrm{gh\#}(\mu)$. In Euclidean signature, the pure co-spinor μ_α can be seen as the complex conjugate of λ^α.

In $D = 10$, the pure spinor constraint is reducible, and has 5 independent components, so a pure spinor has 11 (complex) degrees of freedom. The same thing applies for the constraint on r_α. The antighost singlet cohomology for $D = 10$ super-Yang-Mills sits at $\lambda^3 \theta^5$, and is associated with a Lorentz invariant tensor $T_{(\alpha_1 \alpha_2 \alpha_3)[\beta_1 \ldots \beta_5]}$. There is of course a corresponding tensor $\bar{T}^{(\alpha_1 \alpha_2 \alpha_3)[\beta_1 \ldots \beta_5]}$ with conjugate indices. In ref. [30] this tensor is used to form an invariant integration measure for the pure spinor λ:

$$[d\lambda]\lambda^{\alpha_1}\lambda^{\alpha_2}\lambda^{\alpha_3} \sim \star\bar{T}^{\alpha_1\alpha_2\alpha_3}{}_{\beta_1\ldots\beta_{11}} d\lambda^{\beta_1} \wedge \ldots \wedge d\lambda^{\beta_{11}} \ , \tag{10}$$

where $\star$ refers to dualisation in the β indices. We note that a requirement for this to work is that the number of antisymmetric indices (five) equals the number of irreducible constraints on the spinor, so that the integral is over the full pure spinor space. The corresponding expression with conjugate indices holds for the μ integration, and for the r integration we have

$$[dr] \sim \star\bar{T}^{\alpha_1\alpha_2\alpha_3}{}_{\beta_1\ldots\beta_{11}} \mu_{\alpha_1}\mu_{\alpha_2}\mu_{\alpha_3} \frac{\partial}{\partial r_{\beta_1}} \cdots \frac{\partial}{\partial r_{\beta_{11}}} \ . \tag{11}$$

Using these integration measures, and the ordinary ones for x and θ, we list the dimensions and ghost numbers for the theory after dimensional reduction to D dimensions in table 4. So, the ghost numbers match, and also the dimensions ($\frac{1}{g^2}$ has dimension $D - 4$ in D dimensions), irrespectively of the assignments of $\dim(\mu)$ and $\mathrm{gh\#}(\mu)$.

The λ and μ integrations are non-compact and need regularisation. In ref. [30] this is achieved, following ref. [48], by the insertion of a factor $N = e^{\{\mathcal{Q},\chi\}}$. Since this differs from 1 by a $\mathcal{Q}$-exact term, the regularisation is independent of the choice of the fermion χ. The choice $\chi = -\mu_\alpha \theta^\alpha$ gives $N = e^{-\lambda^\alpha \mu_\alpha - r_\alpha \theta^\alpha}$ and regularises the bosonic integrations at infinity. At the same time, it explains how the term at θ^5 is picked out, this follows after integration over r. N has definite ghost number 0 if $\mathrm{gh\#}(\mu) = -1$ and a dimensionful constant can be avoided in the regulator if $\dim(\mu) = \frac{1}{2}$, so that $\mathrm{gh\#}(r) = 0$ and $\dim(r) = \frac{1}{2}$.

In both the $N = 8$ and $N = 6$ theories in $D = 3$, the naïve measure sits at $\lambda^3 \theta^3$. In analogy with the ten-dimensional case, we need the number of irreducible constraints on the pure spinors to equal the number of θ's. Indeed, the constraints, which in both cases sit

in the vector representation of $so(1,2)$, turn out to be irreducible, which is straightforward to check. The pure spinor spaces are 13- and 9-dimensional, respectively. In both cases, the spinor representation is the tensor product of an $sl(2)$ doublet and a vector under an orthogonal group (in the $N = 8$ case by triality rotation). Letting $\lambda^1 = a + ib$, $\lambda^2 = c + id$, and choosing a set of four orthogonal basis vectors $\{e_1, e_2, e_3, e_4\}$, the general solution to the pure spinor constraint can be parametrised as

$$
\begin{aligned}
a &= \ell e_1 \ , \\
b &= \ell e_2 \ , \\
c &= \ell'(\sin\alpha\cos\beta e_1 + \sin\alpha\sin\beta e_2 + \cos\alpha e_3) \ , \\
d &= \ell'(-\sin\alpha\sin\beta e_1 + \sin\alpha\cos\beta e_2 + \cos\alpha e_4) \ .
\end{aligned}
\tag{12}
$$

There are four real parameters, and the stability group of the parametrisation is $SO(N-4) \subset SO(N)$, so the real dimension of pure spinor space is

$$
4 + \dim(SO(N)) - \dim(SO(N-4)) = 2(2N-3) \ ,
\tag{13}
$$

again giving (complex) dimensions 13 and 9 for the $N = 8$ and $N = 6$ cases, respectively.

We can write the invariant tensors as

$$
\begin{aligned}
&\varepsilon_{abc}(\lambda\gamma^a\theta)(\lambda\gamma^b\theta)(\lambda\gamma^c\theta) \\
&\quad = T_{(A_1\alpha_1, A_2\alpha_2, A_3\alpha_3)[B_1\beta_1, B_2\beta_2, B_3\beta_3]}\lambda^{A_1\alpha_1}\lambda^{A_2\alpha_2}\lambda^{A_3\alpha_3}\theta^{B_1\beta_1}\theta^{B_2\beta_3}\theta^{B_3\beta_3}
\end{aligned}
\tag{14}
$$

in the $N = 8$ case, and as

$$
\begin{aligned}
&\varepsilon_{abc}(\lambda\gamma^a\theta)(\lambda\gamma^b\theta)(\lambda\gamma^c\theta) \\
&\quad = T_{(A_1 i_1, A_2 i_2, A_3 i_3)[B_1 j_1, B_2 j_2, B_3 j_3]}\lambda^{A_1 i_1}\lambda^{A_2 i_2}\lambda^{A_3 i_3}\theta^{B_1 j_1}\theta^{B_2 j_3}\theta^{B_3 j_3}
\end{aligned}
\tag{15}
$$

in the $N = 6$ case (where in both cases the spinor contractions include the $sl(2)$ index, and γ^a are 3-dimensional γ-matrices). The integration measure for a single $N = 8$ pure spinor is then

$$
[d\lambda]\lambda^{A_1\alpha_1}\lambda^{A_2\alpha_2}\lambda^{A_3\alpha_3} \sim \star T^{A_1\alpha_1, A_2\alpha_2, A_3\alpha_3}{}_{B_1\beta_1,\ldots,B_{13}\beta_{13}} d\lambda^{B_1\beta_1} \wedge \ldots \wedge d\lambda^{B_{13}\beta_{13}} \ ,
\tag{16}
$$

and for an $N = 6$ pure spinor

$$
[d\lambda]\lambda^{A_1 i_1}\lambda^{A_2 i_2}\lambda^{A_3 i_3} \sim \star T^{A_1 i_1, A_2 i_2, A_3 i_3}{}_{B_1 j_1,\ldots,B_9 j_9} d\lambda^{B_1 j_1} \wedge \ldots \wedge d\lambda^{B_9 j_9} \ .
\tag{17}
$$

The same expressions apply for the μ integrations, since the "spinor" representations in both cases are self-conjugate. For the r integrations we have

$$
[dr] \sim \star T^{A_1\alpha_1, A_2\alpha_2, A_3\alpha_3}{}_{B_1\beta_1,\ldots,B_{13}\beta_{13}}\mu_{A_1\alpha_1}\mu_{A_2\alpha_2}\mu_{A_3\alpha_3}\frac{\partial}{\partial r_{B_1\beta_1}}\cdots\frac{\partial}{\partial r_{B_{13}\beta_{13}}}
\tag{18}
$$

and

$$
[dr] \sim \star T^{A_1 i_1, A_2 i_2, A_3 i_3}{}_{B_1 j_1,\ldots,B_9 j_9}\mu_{A_1 i_1}\mu_{A_2 i_2}\mu_{A_3 i_3}\frac{\partial}{\partial r_{B_1 j_1}}\cdots\frac{\partial}{\partial r_{B_9 j_9}}
\tag{19}
$$

	$N = 8$		$N = 6$	
	gh#	dim	gh#	dim
d^3x	0	-3	0	-3
$[d\theta]$	0	8	0	6
$[d\lambda]$	10	-5	6	-3
$[d\mu]$	-10	5	-6	3
$[dr]$	-3	-5	-3	-3
total	-3	0	-3	0

Table 5: The dimensions and ghost numbers of the $N = 8$ and $N = 6$ measures.

respectively. Let us examine the dimensions and ghost numbers of the total measures. The analogies of table 4, with $\mathrm{gh\#}(\mu) = -1$ and $\dim(\mu) = \frac{1}{2}$, is given in table 5.

In both cases we get a non-degenerate measure of dimension 0 and ghost number -3, as desired for a conformal theory. Also here, the measures of course have to be regularised in the same way as in ref. [30]. We insert a factor $N = e^{\{\mathcal{Q},\chi\}}$, where $\chi = -\mu_{A\alpha}\theta^{A\alpha}$ for $N = 8$, and $\chi = -\mu_{i\alpha}\theta^{i\alpha}$ for $N = 6$.

To conclude, we have extended our previous manifestly supersymmetric formulation of the $N = 8$ BLG models to the $N = 6$ ABJM models. We have also performed a detailed analysis of the pure spinor constraints and provided proper actions based on non-degenerate measures on non-minimal pure spinor spaces. We hope that these formulations may be helpful in the future, e.g. for the investigation of quantum properties of the models.

Acknowledgments

References

[1] J. Bagger and N. Lambert, *Modeling multiple M2's*, Phys. Rev. **D 75** (2007) 045020 [hep-th/0611108].

[2] A. Gustavsson, *Algebraic structures on parallel M2-branes*, arXiv:0709.1260.

[3] J. Bagger and N. Lambert, *Gauge symmetry and supersymmetry of multiple M2-branes*, Phys. Rev. **D 77** (2008) 065008 [arXiv:0711.0955].

[4] J. Bagger and N. Lambert, *Comments on multiple M2-branes*, JHEP **02** (2008) 105 [arXiv:0712.3738].

[5] G. Papadopoulos, *M2-branes, 3-Lie algebras and Plücker relations*, JHEP **05** (2008) 054 [arXiv:0804.2662].

[6] J.P. Gauntlett and J.B. Gutowski, *Constraining maximally supersymmetric membrane actions*, arXiv:0804.3078.

[7] N. Lambert and D. Tong, *Membranes on an orbifold*, Phys. Rev. Lett. **101** (2008) 041602 [arXiv:0804.1114].

[8] J. Distler, S. Mukhi, C. Papageorgakis and M. Van Raamsdonk, *M2-branes on M-folds*, *JHEP* **05** (2008) 038 [arXiv:0804.1256].

[9] P.-M. Ho and Y. Matsuo, *M5 from M2*, *JHEP* **06** (2008) 105 [arXiv:0804.3629].

[10] I.A. Bandos and P.K. Townsend, *Light-cone M5 and multiple M2-branes*, arXiv:0806.4777; *SDiff gauge theory and the M2 condensate*, arXiv:0808.1583.

[11] U. Gran, B.E.W. Nilsson and C. Petersson, *On relating multiple M2 and D2-branes*, arXiv:0804.1784.

[12] J. Gomis, G. Milanesi and J.G. Russo, *Bagger-Lambert theory for general Lie algebras*, *JHEP* **06** (2008) 075 [arXiv:0805.1012].

[13] S. Benvenuti, D. Rodriguez-Gomez, E. Tonni and H. Verlinde, *N = 8 superconformal gauge theories and M2 branes*, arXiv:0805.1087.

[14] P.-M. Ho, Y. Imamura and Y. Matsuo, *M2 to D2 revisited*, *JHEP* **07** (2008) 003 [arXiv:0805.1202].

[15] O. Aharony, O. Bergman, D.L. Jafferis and J. Maldacena, *N = 6 superconformal Chern-Simons-matter theories, M2-branes and their gravity duals*, arXiv:0806.1218.

[16] D. Gaiotto and E. Witten, *Janus configurations, Chern-Simons couplings, and the theta-angle in N = 4 super Yang-Mills theory*, arXiv:0804.2907.

[17] K. Hosomichi, K.-M. Lee, S. Lee, S. Lee and J. Park, *N = 4 superconformal Chern-Simons theories with hyper and twisted hyper multiplets*, *JHEP* **07** (2008) 091 [arXiv:0805.3662].

[18] M. Benna, I. Klebanov, T. Klose and M. Smedback, *Superconformal Chern-Simons theories and AdS_4/CFT_3 correspondence*, *JHEP* **09** (2008) 072 [arXiv:0806.1519].

[19] T. Nishioka and T. Takayanagi, *On Type IIA Penrose limit and N = 6 Chern-Simons theories*, *JHEP* **08** (2008) 001 [arXiv:0806.3391].

[20] J.A. Minahan and K. Zarembo, *The Bethe ansatz for superconformal Chern-Simons*, *JHEP* **09** (2008) 040 [arXiv:0806.3951].

[21] K. Hosomichi, K.-M. Lee, S. Lee, S. Lee and J. Park, *N = 5, 6 superconformal Chern-Simons theories and M2-branes on orbifolds*, *JHEP* **09** (2008) 002 [arXiv:0806.4977].

[22] J. Bagger and N. Lambert, *Three-algebras and N = 6 Chern-Simons gauge theories*, arXiv:0807.0163.

[23] M. Schnabl and Y. Tachikawa, *Classification of N = 6 superconformal theories of ABJM type*, arXiv:0807.1102.

[24] B.E.W. Nilsson and J. Palmkvist, *Superconformal M2-branes and generalized Jordan triple systems*, arXiv:0807.5134.

[25] N. Lambert, *Lagrangians for multiple M2-branes*, talk given at Strings 2008, CERN, Genève, August 2008, www.cern.ch/strings2008;
J. Maldacena, *N = 6 Chern Simons matter theories, M2 branes and supergravity*, talk given at Strings 2008, CERN, Genève, August 2008, www.cern.ch/strings2008;
S. Mukhi, *Multiple membrane dynamics*, talk given at Strings 2008, CERN, Genève, August 2008, www.cern.ch/strings2008.

[26] M. Cederwall, *N = 8 superfield formulation of the Bagger-Lambert-Gustavsson model*, *JHEP* **09** (2008) 116 [arXiv:0808.3242].

[27] I. Bandos, *NB BLG model in $N = 8$ superfields*, `arXiv:0808.3568`.

[28] A. Mauri and A.C. Petkou, *An $N = 1$ superfield action for M2 branes*, *Phys. Lett.* **B 666** (2008) 527 [`arXiv:0806.2270`].

[29] S. Cherkis and C. Sämann, *Multiple M2-branes and generalized 3-Lie algebras*, *Phys. Rev.* **D 78** (2008) 066019 [`arXiv:0807.0808`].

[30] N. Berkovits, *Pure spinor formalism as an $N = 2$ topological string*, *JHEP* **10** (2005) 089 [`hep-th/0509120`].

[31] A. Gustavsson, *One-loop corrections to Bagger-Lambert theory*, `arXiv:0805.4443`.

[32] J. Bedford and D. Berman, *A note on quantum aspects of multiple membranes*, *Phys. Lett.* **B 668** (2008) 67 [`arXiv:0806.4900`].

[33] N. Berkovits, *Super-Poincaré covariant quantization of the superstring*, *JHEP* **04** (2000) 018 [`hep-th/0001035`].

[34] N. Berkovits, *ICTP lectures on covariant quantization of the superstring*, proceedings of the ICTP Spring School on Superstrings and Related Matters, Trieste, Italy 2002, [`hep-th/0209059`].

[35] B.E.W. Nilsson, *Pure spinors as auxiliary fields in the ten-dimensional supersymmetric Yang-Mills theory*, *Class. and Quant. Grav.* **3** (1986) L41.

[36] P.S. Howe, *Pure spinors lines in superspace and ten-dimensional supersymmetric theories*, *Phys. Lett.* **B 258** (1991) 141 [*Addendum ibid.* **B 259** (1991) 511].

[37] P.S. Howe, *Pure spinors, function superspaces and supergravity theories in ten-dimensions and eleven-dimensions*, *Phys. Lett.* **B 273** (1991) 90.

[38] M. Cederwall, B.E.W. Nilsson and D. Tsimpis, *The structure of maximally supersymmetric Yang-Mills theory: constraining higher-order corrections*, *JHEP* **06** (2001) 034 [`hep-th/0102009`].

[39] M. Cederwall, B.E.W. Nilsson and D. Tsimpis, *$D = 10$ super-Yang-Mills at $O(alpha^2)$*, *JHEP* **07** (2001) 042 [`hep-th/0104236`].

[40] N. Berkovits, *Covariant quantization of the superparticle using pure spinors*, *JHEP* **09** (2001) 016 [`hep-th/0105050`].

[41] M. Cederwall, B.E.W. Nilsson and D. Tsimpis, *Spinorial cohomology and maximally supersymmetric theories*, *JHEP* **02** (2002) 009 [`hep-th/0110069`];
M. Cederwall, *Superspace methods in string theory, supergravity and gauge theory*, `hep-th/0105176`.

[42] M. Movshev and A.S. Schwarz, *On maximally supersymmetric Yang-Mills theories*, *Nucl. Phys.* **B 681** (2004) 324 [`hep-th/0311132`].

[43] M. Cederwall and B.E.W. Nilsson, *Pure spinors and $D = 6$ super-Yang-Mills*, `arXiv:0801.1428`.

[44] M. Cederwall, U. Gran, M. Nielsen and B.E.W. Nilsson, *Manifestly supersymmetric M-theory*, *JHEP* **10** (2000) 041 [`hep-th/0007035`]; *Generalised 11-dimensional supergravity*, `hep-th/0010042`.

[45] M. Cederwall, U. Gran, B.E.W. Nilsson and D. Tsimpis, *Supersymmetric corrections to eleven-dimensional supergravity*, *JHEP* **05** (2005) 052 [`hep-th/0409107`].

[46] P.S. Howe and D. Tsimpis, *On higher-order corrections in M-theory*, *JHEP* **09** (2003) 038 [hep-th/0305129].

[47] P. Fré and P.A. Grassi, *Pure Spinor Formalism for Osp(N|4) backgrounds*, arXiv:0807.0044.

[48] R. Marnelius and M. Ögren, *Symmetric inner products for physical states in BRST quantization*, *Nucl. Phys.* **B 351** (1991) 474.